Combination of Forecasts across Estimation Windows: An Application to Air Travel Demand

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Why Air Travel Demand Forecasts?

1. Forecasting of air travel demand has a long tradition – long-term as well as short-term forecasts.
2. Long-term forecasts are important for decisions, including
   - the expansion of airport facilities,
   - research and development,
   - airplane design and production planning,
   - regional planning of policy makers.
3. Short-term forecasts are important for decisions, including
   - capacity and resource planning of airlines,
   - marketing measures of airlines,
   - capacity and resource planning of airport operators,
   - marketing measures of airport operators.
4. The air travel market is very sensitive to the prevailing business cycle, and it demands frequent updating of forecasts.

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Combination of Forecasts: Basic Ideas

Combining forecasts from different models:

- Since the seminal paper of Bates/Granger (1969), a sizeable literature on the merits of combining forecasts from different models has evolved.
- If forecasts are based on different explanatory variables and/or different assumptions about the relations between the variables, forecast combinations with equal or estimated weights can outperform the individual forecasts.
- In this literature, the different forecasts are typically obtained by estimating a number of alternative models over the same sample period.

Combination of Forecasts: Basic Ideas

Combining forecasts from different models:

Overviews:

- Clemen, R. T. (1989), Combining Forecasts: A Review and Annotated Bibliography, International Journal of Forecasting 5, 559-583.
- Jungmittag, A. (1998), Combination of Forecasts, in: Kotz, S. et al. (Eds.), Encyclopedia of Statistical Sciences, Update Volume 2, New York et al.: John Wiley and Sons, 258-263.
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Combination of Forecasts: Basic Ideas

Combining forecasts across different estimation windows:

- Pesaran/Timmermann (2007) argue that the forecast averaging procedure can be extended to deal with other types of model uncertainty, such as the uncertainty over the size of the estimation window.
- They propose the idea of averaging forecasts from the same model but computed over different estimation windows.
- Using Monte Carlo experiments, they show that this type of forecast averaging reduces the mean square forecast error (MSFE) in many cases when the underlying relations are subject to structural breaks.
- Pesaran/Pick (2011) extend the approach of Pesaran/Timmermann (2007) and apply it to financial market data before and after the recent credit crunch.

Combination of Forecasts: Basic Ideas

Aims of my paper:

- The empirical application in Pesaran/Pick (2011) is limited to a very simple model (random walk with drift) and to one-step ahead forecasts.
- In my paper I use more sophisticated seasonal Box-Jenkins models (SARIMA) — adequate to forecast air travel demand at German airports with distinct seasonal patterns, and
- I compare multi-step forecasts up to 12 months.

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Advantages of the Method

- An attractive feature of this method is that no exact information about the structural break is needed.
- This contrasts with the conventional approach of assessing or estimating the break points, and then basing the forecasts on the post-breaks observations.
- However, it is not always optimal to base forecasts only on the post-break data.
- Using pre-break data biases the forecast, but at the same time it reduces the forecast error variance.
- The overall effect of using pre-break data depends on the the size and the point of the break — and it is hard to assess the size of the break since it involves estimating the model over the pre- and post break periods.
- If the distance to the break is short, the post-break parameters are likely to be poorly estimated.

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Forecast Averaging across Estimation Windows: Notation and Basic Approach

- Consider the sample $\{y_t, x_t\}_{t=T_i+1}^{T}$, with $0 \leq T_i < T$, which yields an observation window of size $W_i = T - T_i$.
- Then, the fraction of observations in a single window is $w_i = (T - T_i)/T$.
- The estimation process could start with a minimum window $\{y_t, x_t\}_{t=T_{\min}+1}^{T}$ of size $w_{\min} = (T - T_{\min})/T$.
- From $w_{\min}$ other larger windows can be considered.
- More specifically, it is

$$w_i = w_{\min} + \left(\frac{i-1}{m-1}\right)(1 - w_{\min}), \text{ for } i = 1, 2, \ldots, m,$$

so that $w_i \in [w_{\min}, 1]$. The number of estimation windows is m.

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Average Window Forecast

- The average window forecast is defined by

$$\hat{y}_{m,T+1} = \frac{1}{m}\sum_{i=1}^{m} \hat{y}_{T+1}(w_i),$$

  where $\hat{y}_{T+1}(w_i)$ is the forecast from an estimation window w_i, and forecasts from all windows are given equal weights.
- The objective of my analysis is to compare single-window forecasts and the average window forecasts.
- In recursive estimation, the single window can be an expanding or a rolling window, and average window forecasts can be obtained by averaging over sub-windows within an given expanding or rolling window.
- Therefore, the average window forecast procedure is not an alternative to rolling forecasts.

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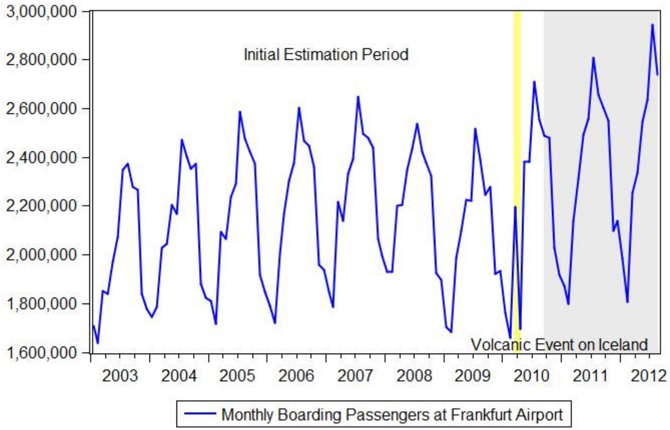
Boarding Passengers per Month at Frankfurt Airport, January 2003 until August 2012



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Specification of the SARIMA-Model for the Whole Estimation Period

Seasonal autoregressive integrated moving average (SARIMA) model

examines the year-to-year relationships for each month (in our case) of a time series (Box/Jenkins/Reinsel, 1994), thus capturing the seasonal relationship between observations for the same month (Y_t and Y_{t-12}) in successive years.

The error terms of the model are assumed to have a zero mean, constant variance, and to be serially independent.

Very popular class of models to forecast air passengers.

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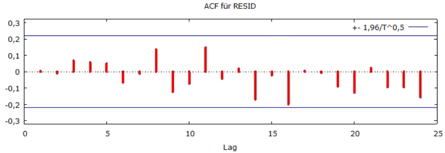
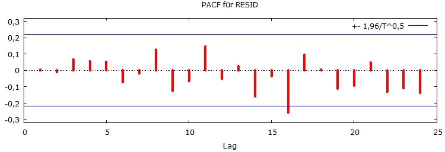
Specification of the SARIMA-Model for the Whole Estimation Period

Dependent Variable: DLOG(FFM,0,12)
Sample: 2004M02 2010M09
Included observations: 80
Convergence achieved after 9 iterations
MA Backcast: 2003M01 2004M01

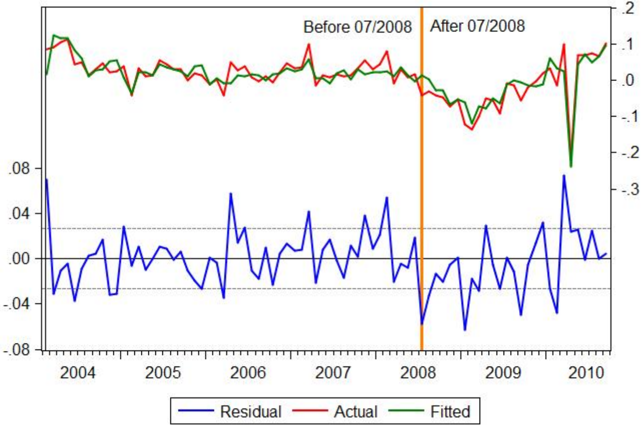
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003266	0.006687	0.488418	0.6267
D(VOLCAN,0,12)	-0.256077	0.022822	-11.22079	0.0000
AR(1)	0.871729	0.065142	13.38201	0.0000
MA(1)	-0.430945	0.125713	-3.428016	0.0010
SMA(12)	-0.877454	0.022908	-38.30357	0.0000

R-squared	0.787406	Mean dependent var	0.010958
Adjusted R-squared	0.776068	S.D. dependent var	0.056307
S.E. of regression	0.026645	Akaike info criterion	-4.351952
Sum squared resid	0.053248	Schwarz criterion	-4.203075
Log likelihood	179.0781	Hannan-Quinn criter.	-4.292263
F-statistic	69.44642	Durbin-Watson stat	1.899966
Prob(F-statistic)	0.000000		





Specification of the SARIMA-Model for the Whole Estimation Period

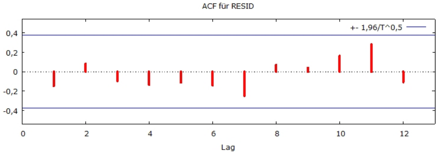


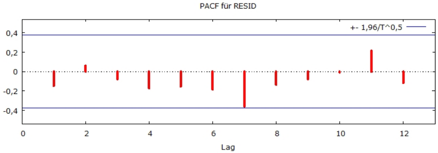
Re-Estimation of the SARIMA-Model for the Post-Break Period

Dependent Variable: DLOG(FFM,0,12)
Sample: 2008M07 2010M09
Included observations: 27
Convergence achieved after 530 iterations
MA Backcast: 2007M06 2008M06

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	89.78074	46888.04	0.001915	0.9985
D(VOLCAN,0,12)	-0.265630	0.024780	-10.71958	0.0000
AR(1)	0.999893	0.056051	17.83885	0.0000
MA(1)	-0.794173	0.175070	-4.536316	0.0002
SMA(12)	-0.942646	0.036277	-25.98493	0.0000

R-squared	0.937988	Mean dependent var	-0.023555
Adjusted R-squared	0.926714	S.D. dependent var	0.075408
S.E. of regression	0.020414	Akaike info criterion	-4.779603
Sum squared resid	0.009168	Schwarz criterion	-4.539633
Log likelihood	69.52464	Hannan-Quinn criter.	-4.708247
F-statistic	83.19318	Durbin-Watson stat	2.229079
Prob(F-statistic)	0.000000		





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Design of the Forecasting Exercise

- The comparison of the performance of the different forecast methods is carried out for short-term to medium-term forecast horizons, 1 to 12 months ahead.
- The evaluation is based on recursive forecasts that involve an average of the respective horizon forecasts over all 12 recursive windows.
- The one step ahead forecasts are starting with the forecast for 2010(10) based on an estimation window up to 2010(9), the second forecast is for 2010(11) based on an estimation window up to 2010(10), etc., the 12th forecast for 2011(9) is then based on an estimation window up to 2011(8).
- Similarly, 12-period-ahead forecasts are carried out for 12 different recursive windows. The forecast for 2011(9) is based on an window up to 2010(9), whereas the last 12 step ahead forecast is carried out for 2012(8) based on an estimation window until 2011(8).

Methods Used in the Forecasting Exercise

Baseline forecasts

(i) using an expanding window of the observations after the structural break.

Average window forecasts

with a minimum number of observations $W_{\min} = 27$ months and $m = 5$ sub-windows within

(ii) a rolling window of length $W_a = 80$ months and

(iii) an expanding window.

Expanding window

(iv) over the whole observation period.

Methods Used in the Forecasting Exercise

Single rolling windows

of size

(v) $W_{\min} = 27$ months,

(vi) $\bar{W} = W_a(1/5 + 2/5 + \ldots + 5/5)/5 = 48$ months, and

(vii) $W_a = 80$ months.

Measuring the Forecast Performance

Mean absolute forecast error

$$MAFE = \frac{1}{n}\sum_{1}^{n} |\hat{y}_{T+i|T} - y_{T+i}|,$$

where n is the number of forecasts, and i is the step size of a forecast.

Root mean square forecast error

$$RMSFE = \sqrt{\frac{1}{n}\sum_{1}^{n} \left(\hat{y}_{T+i|T} - y_{T+i}\right)^2},$$

where n is again the number of forecasts, and i is the step size of a forecast.

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Performance of the Alternative Forecasts: MAFE

Steps ahead	Post-break	Average Windows		Expanding	Rolling Windows		
		Rolling	Expanding	Windows	$W_{min} = 27$	$W = 48$	$W_{l} = 80$
1 month	64277	49339	47614	43396	60794	51210	44854
2 months	79597	57010	56559	54164	83141	67610	54828
3 months	83537	55388	56437	61410	75559	69548	57995
4 months	73579	73159	73331	84527	77351	91605	83210
5 months	87276	78934	77843	98291	88956	100092	92956
6 months	108776	75016	78903	109425	106511	103383	100409
7 months	101596	65773	70827	117793	92595	117097	107684
8 months	123635	70153	72195	116980	111720	123557	108958
9 months	146187	70084	64992	117594	125096	121806	102977
10 months	153646	57786	50345	117040	135178	105738	99431
11 months	159308	45988	42217	123194	144905	89125	104865
12 months	173808	46803	50210	127939	164074	75914	110093

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Performance of the Alternative Forecasts: Relative MAFE

Steps ahead	Post-break	Average Windows		Expanding	Rolling Windows		
		Rolling	Expanding	Windows	$W_{min} = 27$	$W = 48$	$W_{i} = 80$
1 month	1	0.768	0.741	0.675	0.946	0.797	0.698
2 months	1	0.716	0.711	0.680	1.045	0.849	0.689
3 months	1	0.663	0.676	0.735	0.904	0.833	0.694
4 months	1	0.994	0.997	1.149	1.051	1.245	1.131
5 months	1	0.904	0.892	1.126	1.019	1.147	1.065
6 months	1	0.690	0.725	1.006	0.979	0.950	0.923
7 months	1	0.647	0.697	1.159	0.911	1.153	1.060
8 months	1	0.567	0.584	0.946	0.904	0.999	0.881
9 months	1	0.479	0.445	0.804	0.856	0.833	0.704
10 months	1	0.376	0.328	0.762	0.880	0.688	0.647
11 months	1	0.289	0.265	0.773	0.910	0.559	0.658
12 months	1	0.269	0.289	0.736	0.944	0.437	0.633

Performance of the Alternative Forecasts: RMSFE

Steps ahead	Post-break	Average Windows		Expanding	Rolling Windows		
		Rolling	Expanding	Windows	$W_{min} = 27$	$W = 48$	W[illegible] $= 80$
1 month	90816	64098	61128	61668	84312	67355	60563
2 months	103286	73672	71984	77775	103306	96760	75277
3 months	112188	74856	75890	90780	104359	99615	85043
4 months	99680	84139	84097	107060	101631	123152	102443
5 months	125019	92025	90159	115322	128209	129041	109101
6 months	156889	90692	92873	121611	154600	128613	111330
7 months	136701	80408	82844	126842	132205	139295	117477
8 months	158673	80088	82336	126465	152381	135402	118884
9 months	175420	87329	83083	129755	161870	137623	120016
10 months	201446	73081	68540	126337	194643	122990	114611
11 months	209968	60674	61659	131191	204749	102250	114845
12 months	230733	68609	70230	137322	226936	100881	119217

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Performance of the Alternative Forecasts: Relative RMSFE

Steps ahead	Post-break	Average Windows		Expanding	Rolling Windows		
		Rolling	Expanding	Windows	$W_{min} = 27$	$W = 48$	$W_{\text{[illegible]}} = 80$
1 month	1	0.706	0.673	0.679	0.928	0.742	0.667
2 months	1	0.713	0.697	0.753	1.000	0.937	0.729
3 months	1	0.667	0.676	0.809	0.930	0.888	0.758
4 months	1	0.844	0.844	1.074	1.020	1.235	1.028
5 months	1	0.736	0.721	0.922	1.026	1.032	0.873
6 months	1	0.578	0.592	0.775	0.985	0.820	0.710
7 months	1	0.588	0.606	0.928	0.967	1.019	0.859
8 months	1	0.505	0.519	0.797	0.960	0.853	0.749
9 months	1	0.498	0.474	0.740	0.923	0.785	0.684
10 months	1	0.363	0.340	0.627	0.966	0.611	0.569
11 months	1	0.289	0.294	0.625	0.975	0.487	0.547
12 months	1	0.297	0.304	0.595	0.984	0.437	0.517

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Some Conclusions

- I analyzed whether forecasts of air travel demand — which is very sensitive to business cycles and perhaps structural breaks — can be improved by combining forecasts across different estimation windows.
- One result is very obvious: Nearly every thing provides better forecasts than just using the observations from the post-break period.
- However, the average window forecasts mostly outperform the alternative single window forecasts.
- The relative performance is the stronger, the longer the forecast horizon.

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Some Conclusions

- More generally, averaging of forecasts over different estimation windows offers a simple approach to generating forecasts that are reasonably robust to structural breaks of unknown dates and sizes.
- Therefore, the average windows approach surely would be fruitful for other market or sales forecast tasks.
- The approach also will likely improve the forecast accuracy of other models, e.g. with independent explanatory variables — but this is left for future research.

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