Einige Lösungen zu Mathe 2/ Übung 2

Teil 1

In[2]:=
```
(* 1 *)
```

In[3]:=
```
A = {{1, 2}, {3, 4}}
```

Out[3]= {{1, 2}, {3, 4}}

In[4]:=
```
A // MatrixForm
```

Out[4]//MatrixForm=

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

In[5]:=
```
B = {{0, 1}, {-1, 0}}
```

Out[5]= {{0, 1}, {-1, 0}}

In[6]:=
```
B // MatrixForm
```

Out[6]//MatrixForm=

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

In[7]:=
```
-2 A // MatrixForm
```

Out[7]//MatrixForm=

$$\begin{pmatrix} -2 & -4 \\ -6 & -8 \end{pmatrix}$$

In[8]:=
```
A - B // MatrixForm
```

Out[8]//MatrixForm=

$$\begin{pmatrix} 1 & 1 \\ 4 & 4 \end{pmatrix}$$

In[9]:=
```
A.B // MatrixForm
```

Out[9]//MatrixForm=

$$\begin{pmatrix} -2 & 1 \\ -4 & 3 \end{pmatrix}$$

In[10]:=
```
B.A // MatrixForm
```

Out[10]//MatrixForm=

$$\begin{pmatrix} 3 & 4 \\ -1 & -2 \end{pmatrix}$$

In[11]:=

```
(* 2 *)
```

In[12]:=
```
(* a *)
```

In[13]:=
```
Y = {{2, I}, {-I, 3}}
```

Out[13]= {{2, ⅈ}, {-ⅈ, 3}}

In[14]:= `Z = Inverse[Y]`

Out[14]= $\left\{\left\{\frac{3}{5}, -\frac{i}{5}\right\}, \left\{\frac{i}{5}, \frac{2}{5}\right\}\right\}$

In[15]:=

In[16]:= `gl = {i1, i2} == Y.{u1, u2}`

Out[16]= {i1, i2} == {2 u1 + i u2, - i u1 + 3 u2}

In[17]:= `Solve[gl, {u1, i1}] // Expand`

Out[17]= {{u1 → i i2 - 3 i u2, i1 → 2 i i2 - 5 i u2}}

In[18]:= `(* daraus liest man ab: *)`

In[19]:= `K = {{-3 I, I}, {-5 I, 2 I}}`

Out[19]= {{-3 i, i}, {-5 i, 2 i}}

In[20]:=

`(* b *)`

In[21]:= `Kgesamt = K.K`

Out[21]= {{-4, 1}, {-5, 1}}

In[22]:= `% // MatrixForm`

Out[22]//MatrixForm=

$$\begin{pmatrix} -4 & 1 \\ -5 & 1 \end{pmatrix}$$

Teil 2

In[23]:= `(* 4 *)`

In[24]:=

In[25]:= `A = {{3, 1}, {-1, 2}}`

Out[25]= {{3, 1}, {-1, 2}}

In[26]:= `% // MatrixForm`

Out[26]//MatrixForm=

$$\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

In[27]:= `c = {2, 1}`

Out[27]= {2, 1}

In[28]:= `{x, y} = Inverse[A].c`

Out[28]= $\left\{\frac{3}{7}, \frac{5}{7}\right\}$

In[29]:=
```
x
```
Out[29]= $\frac{3}{7}$

In[30]:=
```
y
```
Out[30]= $\frac{5}{7}$

In[31]:=
```
(* 5 *)
```

In[32]:=
```
(* a *)
```

In[33]:=
```
A = {{s, 1}, {-1, 2}}
```
Out[33]= {{s, 1}, {-1, 2}}

In[34]:=
```
% // MatrixForm
```
Out[34]//MatrixForm=
$$\begin{pmatrix} s & 1 \\ -1 & 2 \end{pmatrix}$$

In[35]:=
```
Solve[Det[A] == 0, s]
```
Out[35]= $\left\{\left\{s \to -\frac{1}{2}\right\}\right\}$

In[36]:=
```
(* b *)
```

In[37]:=
```
s = 2
```
Out[37]= 2

In[38]:=
```
A
```
Out[38]= {{2, 1}, {-1, 2}}

In[39]:=
```
x = {1, 2}
```
Out[39]= {1, 2}

In[40]:=
```
y = A.x
```
Out[40]= {4, 3}

In[41]:=
```
x = {0, -1}
```
Out[41]= {0, -1}

In[42]:=
```
y = A.x
```
Out[42]= {-1, -2}

In[43]:=

Teil 3

In[44]:=
```
(* 7 *)
```

In[45]:=
```
A = {{1, 3, 2}, {0, 1, 0}, {2, 0, 1}}
```
Out[45]= {{1, 3, 2}, {0, 1, 0}, {2, 0, 1}}

In[46]:=
```
(* die Inverse, also Ergebnis für a & c *)
```

In[47]:=
```
Ainv = Inverse[A]
```
Out[47]= $\left\{\left\{-\frac{1}{3}, 1, \frac{2}{3}\right\}, \{0, 1, 0\}, \left\{\frac{2}{3}, -2, -\frac{1}{3}\right\}\right\}$

In[48]:=
```
Ainv // MatrixForm
```
Out[48]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{3} & 1 & \frac{2}{3} \\ 0 & 1 & 0 \\ \frac{2}{3} & -2 & -\frac{1}{3} \end{pmatrix}$$

In[49]:=
```
(* b *)
```

In[50]:=
```
Det[A]
```
Out[50]= −3

In[51]:=
```
(* d *)
```

In[52]:=
```
c = {2, 1, 0}
```
Out[52]= {2, 1, 0}

In[53]:=
```
(* i *)
```

In[54]:=
```
{x, y, z} = Ainv.c
```
Out[54]= $\left\{\frac{1}{3}, 1, -\frac{2}{3}\right\}$

In[55]:=
```
x
```
Out[55]= $\frac{1}{3}$

In[56]:=
```
y
```
Out[56]= 1

In[57]:=
```
z
```
Out[57]= $-\frac{2}{3}$

In[58]:=
```
(* ii *)
```

In[59]:=
```
A // MatrixForm
```
Out[59]//MatrixForm=

$$\begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

In[60]:=
```
AT = A // Transpose
```
Out[60]= {{1, 0, 2}, {3, 1, 0}, {2, 0, 1}}

In[61]:= **AT // MatrixForm**

Out[61]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

In[62]:= **B1T = {C, {3, 1, 0}, {2, 0, 1}}**

Out[62]= {{2, 1, 0}, {3, 1, 0}, {2, 0, 1}}

In[63]:= **B1 = B1T // Transpose**

Out[63]= {{2, 3, 2}, {1, 1, 0}, {0, 0, 1}}

In[64]:= **B1 // MatrixForm**

Out[64]//MatrixForm=

$$\begin{pmatrix} 2 & 3 & 2 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[65]:=

$$\begin{pmatrix} 2 & 3 & 2 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Out[65]= {{2, 3, 2}, {1, 1, 0}, {0, 0, 1}}

In[66]:= **x = Det[B1] / Det[A]**

Out[66]= $\frac{1}{3}$

In[67]:=

(* 9 *)

In[68]:= **T = {{0, 0, 1}, {0, 0, 2}, {1, 2, 0}}**

Out[68]= {{0, 0, 1}, {0, 0, 2}, {1, 2, 0}}

In[69]:= **T // MatrixForm**

Out[69]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

In[70]:= **(* a *)**

In[71]:= **Eigenvalues[T]**

Out[71]= $\left\{-\sqrt{5}, \sqrt{5}, 0\right\}$

In[72]:= **(* b *)**

In[73]:= **es = Eigensystem[T]**

Out[73]= $\left\{\left\{-\sqrt{5}, \sqrt{5}, 0\right\}, \left\{\left\{-\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 1\right\}, \left\{\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 1\right\}, \{-2, 1, 0\}\right\}\right\}$

In[74]:= **ew = es[[1]]**

Out[74]= $\left\{-\sqrt{5}, \sqrt{5}, 0\right\}$

In[75]:= **ev = es[[2]]**

Out[75]= $\left\{\left\{-\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 1\right\}, \left\{\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 1\right\}, \{-2, 1, 0\}\right\}$

In[76]:= **(* der erste Eigenvektor; muss zum ersten Eigenwert gehören: *)**

In[77]:= **ev1 = ev[[1]]**

Out[77]= $\left\{-\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 1\right\}$

In[78]:= **ew1 = ew[[1]]**

Out[78]= $-\sqrt{5}$

In[79]:= **(* Probe:)**

In[79]:= **T.ev1 == ew1 * ev1**

Out[79]= True

In[80]:= **T.ev1**

Out[80]= $\left\{1, 2, -\sqrt{5}\right\}$

In[81]:= **ew1 * ev1**

Out[81]= $\left\{1, 2, -\sqrt{5}\right\}$

In[82]:=

(* c *)

In[83]:= **(* Einheits-Eigenvektoren *)**

In[84]:= **ev1n = ev1 / Norm[ev1]**

Out[84]= $\left\{-\frac{1}{\sqrt{10}}, -\sqrt{\frac{2}{5}}, \frac{1}{\sqrt{2}}\right\}$

In[85]:= **ev2 = ev[[2]]**

Out[85]= $\left\{\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 1\right\}$

In[86]:= **ev3 = ev[[3]]**

Out[86]= {-2, 1, 0}

In[87]:= **ev2n = ev2 / Norm[ev2]**

Out[87]= $\left\{\frac{1}{\sqrt{10}}, \sqrt{\frac{2}{5}}, \frac{1}{\sqrt{2}}\right\}$

In[88]:= **ev3n = ev3 / Norm[ev3]**

Out[88]= $\left\{-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right\}$

In[89]:= `Cross[ev1n, ev2n].ev3n`

Out[89]= 1

Die Vektoren bilden also in der Reihenfolge 1,2,3 ein Rechtssytem und spannen das gesuchte neue Koordinatensystem S' auf; sie werden jetzt als Zeilen der Drehmatrix, welche von S nach S' transformiert, benutzt

In[91]:= `d = {ev1n, ev2n, ev3n}`

Out[91]= $\left\{\left\{-\frac{1}{\sqrt{10}}, -\sqrt{\frac{2}{5}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{10}}, \sqrt{\frac{2}{5}}, \frac{1}{\sqrt{2}}\right\}, \left\{-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right\}\right\}$

`(* i *)`

In[92]:= `Det[d]`

Out[92]= 1

`(* ii *)`

In[95]:= `invd = Inverse[d]`

Out[95]= $\left\{\left\{-\frac{1}{\sqrt{10}}, \frac{1}{\sqrt{10}}, -\frac{2}{\sqrt{5}}\right\}, \left\{-\sqrt{\frac{2}{5}}, \sqrt{\frac{2}{5}}, \frac{1}{\sqrt{5}}\right\}, \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\}\right\}$

In[96]:= `transd = Transpose[d]`

Out[96]= $\left\{\left\{-\frac{1}{\sqrt{10}}, \frac{1}{\sqrt{10}}, -\frac{2}{\sqrt{5}}\right\}, \left\{-\sqrt{\frac{2}{5}}, \sqrt{\frac{2}{5}}, \frac{1}{\sqrt{5}}\right\}, \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\}\right\}$

In[97]:= `invd == transd`

Out[97]= True