Einige Lösungen zu Mathe 2/ Übung 3

In[99]:=
```
Remove["Global`*"]
(* 1 *)
(* a *)
(* s. Formelsammlung *)
(* b *)
```

In[100]:=
```
Series[Cos[x], {x, Pi / 2, 5}]
```

Out[100]= $-\left(x-\frac{\pi}{2}\right)+\frac{1}{6}\left(x-\frac{\pi}{2}\right)^3-\frac{1}{120}\left(x-\frac{\pi}{2}\right)^5+O\left[x-\frac{\pi}{2}\right]^6$

In[101]:=
```
Series[Cos[x], {x, Pi / 2, 6}]
```

Out[101]= $-\left(x-\frac{\pi}{2}\right)+\frac{1}{6}\left(x-\frac{\pi}{2}\right)^3-\frac{1}{120}\left(x-\frac{\pi}{2}\right)^5+O\left[x-\frac{\pi}{2}\right]^7$

In[102]:=
```
Series[Cos[x], {x, Pi / 2, 7}]
```

Out[102]= $-\left(x-\frac{\pi}{2}\right)+\frac{1}{6}\left(x-\frac{\pi}{2}\right)^3-\frac{1}{120}\left(x-\frac{\pi}{2}\right)^5+\frac{\left(x-\frac{\pi}{2}\right)^7}{5040}+O\left[x-\frac{\pi}{2}\right]^8$

```
(* explizit über die Formel für das Taylorpolynom: *)
```

In[103]:=
```
f[x_] = Cos[x]
```

Out[103]= Cos[x]

In[105]:=
```
x0 = Pi / 2
```

Out[105]= $\frac{\pi}{2}$

```
i = 1;
(* 1. Ableitung von f(x) an der Stelle x=x0 *)
```

In[110]:=
```
D[f[x], {x, i}] /. x -> x0
```

Out[110]= 1

In[108]:=
```
Sum[(D[f[x], {x, i}] /. x -> x0) / i! * (x - x0)^i, {i, 0, 7}]
```

Out[108]= $\frac{\pi}{2}-x+\frac{1}{6}\left(-\frac{\pi}{2}+x\right)^3-\frac{1}{120}\left(-\frac{\pi}{2}+x\right)^5+\frac{\left(-\frac{\pi}{2}+x\right)^7}{5040}$

```
(* 2 *)
(* s. Formelsammlung *)
```

```
(* 3 *)

(* √17 = √16+1 = 4√1 + 1/16 ;
  mit  g(x) = 4 √1+x   suchen wir also  g(1/16);
  wir entwickeln dazu g(x) um x0 =
  0  und berechnen damit näherungsweise g(1/16);
  die Entwicklung steht in der Formelsammlung *)
```

In[111]:= `g[x_] = 4 Sqrt[1 + x]`

Out[111]= $4\sqrt{1+x}$

In[126]:= `tp3g[x_] = Series[g[x], {x, 0, 3}] // Normal`

Out[126]= $4 + 2x - \frac{x^2}{2} + \frac{x^3}{4}$

In[127]:= `w17 = tp3g[1 / 16]`

Out[127]= $\frac{67553}{16384}$

In[120]:= `% // N`

Out[120]= 4.12311

In[121]:= `N[w17, 20]`

Out[121]= 4.1231079101562500000

In[128]:= `tp5g[x_] = Series[g[x], {x, 0, 5}] // Normal`

Out[128]= $4 + 2x - \frac{x^2}{2} + \frac{x^3}{4} - \frac{5x^4}{32} + \frac{7x^5}{64}$

In[129]:= `w17 = tp5g[1 / 16]`

Out[129]= $\frac{276696935}{67108864}$

In[130]:= `% // N`

Out[130]= 4.12311

In[125]:= `N[w17, 20]`

Out[125]= 4.1231056302785873413

```
(* die ersten 4 Nachkommastellen sind also ok *)

(* Konvergenzradius r = 1, s. Formelsammlung,
   d.h. bis g(1)= √32  können wir durch eine höhere Ordnung
   eine bessere Approximation erwarten;
   Gegenbeispiel: √35 = √16 + 19 = 4√1+19/16 = g(19/16) : *)
```

In[132]:= `N[tp3g[19 / 16], 20]`

Out[132]= 6.0885620117187500000

In[133]:= `N[tp5g[19 / 16], 20]`

Out[133]= 6.0361297875642776489

In[135]:= `tp7g[x_] = Series[g[x], {x, 0, 7}] // Normal`

Out[135]= $4+2x-\frac{x^2}{2}+\frac{x^3}{4}-\frac{5x^4}{32}+\frac{7x^5}{64}-\frac{21x^6}{256}+\frac{33x^7}{512}$

In[136]:= `N[tp7g[19 / 16], 20]`

Out[136]= 6.0207261149407713674

In[139]:= `tp20g[x_] = Series[g[x], {x, 0, 20}] // Normal;`

In[140]:= `N[tp20g[19 / 16], 20]`

Out[140]= 5.7065996920508793953

In[141]:= `tp30g[x_] = Series[g[x], {x, 0, 30}] // Normal;`

In[142]:= `N[tp30g[19 / 16], 20]`

Out[142]= 5.2766395545629998173

In[134]:= $N\left[\sqrt{35}, 20\right]$

Out[134]= 5.9160797830996160426

(* 5 *)

In[143]:= `f[x_] = Exp[-(x + 1)]`

Out[143]= e^{-1-x}

In[144]:= `g[x_] = Log[1 + x]`

Out[144]= Log[1 + x]

(* a *)

In[146]:= `tpf[x_] = Series[f[x], {x, 0, 2}] // Normal`

Out[146]= $\frac{1}{e}-\frac{x}{e}+\frac{x^2}{2e}$

In[147]:= `tpg[x_] = Series[g[x], {x, 0, 2}] // Normal`

Out[147]= $x-\frac{x^2}{2}$

In[149]:= `gl = tpf[x] == tpg[x]`

Out[149]= $\frac{1}{e}-\frac{x}{e}+\frac{x^2}{2e}=x-\frac{x^2}{2}$

In[150]:= `Solve[gl, x]`

Out[150]= $\left\{\left\{x\to\frac{2+2e-\sqrt{-4+4e^2}}{2+2e}\right\},\left\{x\to\frac{2+2e+\sqrt{-4+4e^2}}{2+2e}\right\}\right\}$

In[151]:= `% // N`

Out[151]= `{{x → 0.320208}, {x → 1.67979}}`

In[152]:= `x1 = 0.320208`

Out[152]= `0.320208`

In[153]:= `x2 = 1.67979`

Out[153]= `1.67979`

`(* Probe !!! *)`

In[154]:= `f[x1]`

Out[154]= `0.26708`

In[155]:= `g[x1]`

Out[155]= `0.277789`

In[156]:= `f[x2]`

Out[156]= `0.0685776`

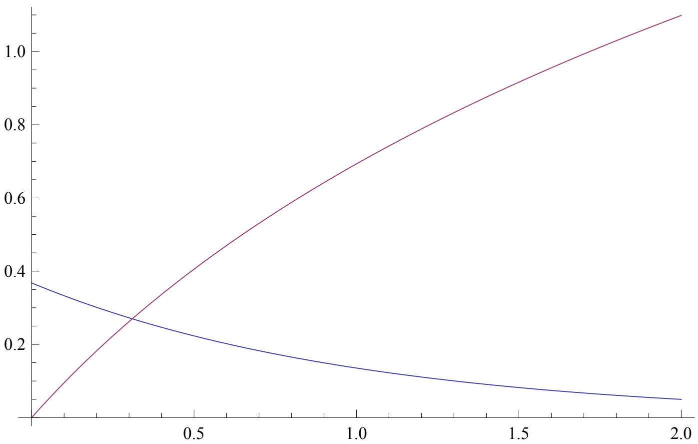
In[157]:= `g[x2]`

Out[157]= `0.985738`

`(* nur x1 kommt also Lösung des Originalproblems in Betracht,`
`wie auch die Grafik zeigt: *)`

In[158]:= `Plot[{f[x], g[x]}, {x, 0, 2}]`

Out[158]=



`(* b *)`

`(* Newton-Verfahren: *)`

In[159]:= `h[x_] = f[x] - g[x]`

Out[159]= e^{-1-x} `- Log[1 + x]`

`(* die Iterationsformel für das N.-V.: *)`

In[160]:= `x[n_] := x[n - 1] - h[x[n - 1]] / h'[x[n - 1]]`

`(* Startwert: *)`

In[161]:= `x[0] = x1`

Out[161]= 0.320208

In[162]:= `x[1]`

Out[162]= 0.309755

In[163]:= `x[2]`

Out[163]= 0.3098

In[164]:= `x[3]`

Out[164]= 0.3098

```
(* eingebautes Newton-Verfahren: *)
```

In[166]:= `FindRoot[f[x] == g[x], {x, x1}, WorkingPrecision → 20]`

Out[166]= {x → 0.30979958580415047767}

```
(* 7 *)

(* a *)

(* über die Substitution u = -3 x; x = x0 = 0 ⟹ u = 0 = u0 *)
```

In[167]:= `g[u_] = Exp[u]`

Out[167]= e^u

```
tp5g = Series[g[u], {u, 0, 5}] // Normal (* s. Formels. *)
```

Out[173]= $1 + u + \frac{u^2}{2} + \frac{u^3}{6} + \frac{u^4}{24} + \frac{u^5}{120}$

In[174]:= `tp5f = tp5g /. (u → -3 x)`

Out[174]= $1 - 3x + \frac{9x^2}{2} - \frac{9x^3}{2} + \frac{27x^4}{8} - \frac{81x^5}{40}$

```
(* das ist das Taylorp. 5. O. für f *)


(* b *)
```

In[175]:= `f[t_] = Sin[t^2]`

Out[175]= $\mathrm{Sin}[t^2]$

```
(* Taylorp. über die Substitution u = t^2; t = t0 = 0 ⟹ u = 0 = u0 *)
```

In[179]:= `g[u_] = Sin[u]`

Out[179]= Sin[u]

In[186]:= `tpg = Series[g[u], {u, 0, 3}] // Normal`

Out[186]= $u - \frac{u^3}{6}$

In[187]:=
```
tpf[t_] = tpg /. u → t^2
```
Out[187]= $t^2 - \frac{t^6}{6}$

In[188]:=
```
Integrate[tpf[t], {t, 0, 1}] // N
```
Out[188]= 0.309524

In[189]:=
```
tpg = Series[g[u], {u, 0, 5}] // Normal
```
Out[189]= $u - \frac{u^3}{6} + \frac{u^5}{120}$

In[190]:=
```
tpf[t_] = tpg /. u → t^2
```
Out[190]= $t^2 - \frac{t^6}{6} + \frac{t^{10}}{120}$

In[191]:=
```
Integrate[tpf[t], {t, 0, 1}] // N
```
Out[191]= 0.310281

In[192]:=
```
tpg = Series[g[u], {u, 0, 7}] // Normal
```
Out[192]= $u - \frac{u^3}{6} + \frac{u^5}{120} - \frac{u^7}{5040}$

In[193]:=
```
tpf[t_] = tpg /. u → t^2
```
Out[193]= $t^2 - \frac{t^6}{6} + \frac{t^{10}}{120} - \frac{t^{14}}{5040}$

In[194]:=
```
Integrate[tpf[t], {t, 0, 1}] // N
```
Out[194]= 0.310268

```
(* die ersten 3 Nachkommastellen sind also ok *)
```