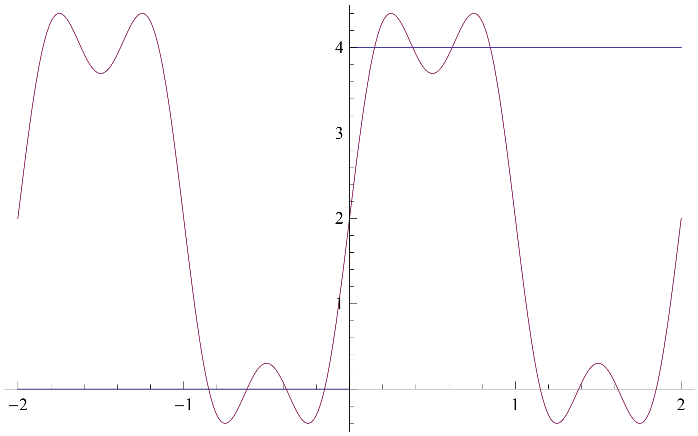
```
(* Mathe Vertiefung, Übung 4 *)

(* a *)

f[t_] := 4 UnitStep[t]

(* die Periode: *)

p = 2;

ω = 2 Pi / p
```

π

```
(* Fourier-Reihe bis zur 3. Ordnung, also bis zur 2. Oberschwingung *)

fr[t_, 3] = FourierTrigSeries[f[t], t, 3, FourierParameters → {1, ω}]
```

$$2 + \frac{8\,\mathrm{Sin}[\pi\,t]}{\pi} + \frac{8\,\mathrm{Sin}[3\,\pi\,t]}{3\,\pi}$$

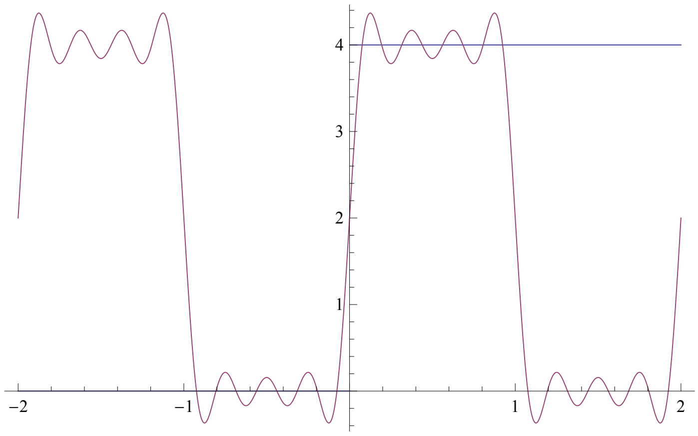
```
Plot[{f[t], fr[t, 3]}, {t, -2, 2}]
```



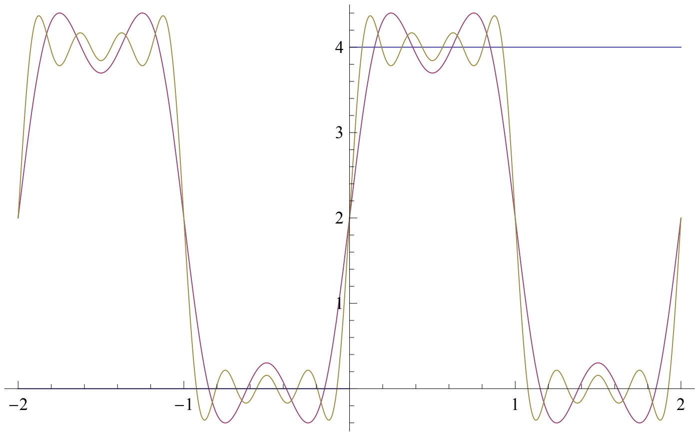

```
(* Fourier-Reihe bis zur 7. Ordnung, also bis zur 6. Oberschwingung *)

fr[t_, 7] = FourierTrigSeries[f[t], t, 7, FourierParameters → {1, ω}]
```

$$2 + \frac{8\,\mathrm{Sin}[\pi\,t]}{\pi} + \frac{8\,\mathrm{Sin}[3\,\pi\,t]}{3\,\pi} + \frac{8\,\mathrm{Sin}[5\,\pi\,t]}{5\,\pi} + \frac{8\,\mathrm{Sin}[7\,\pi\,t]}{7\,\pi}$$

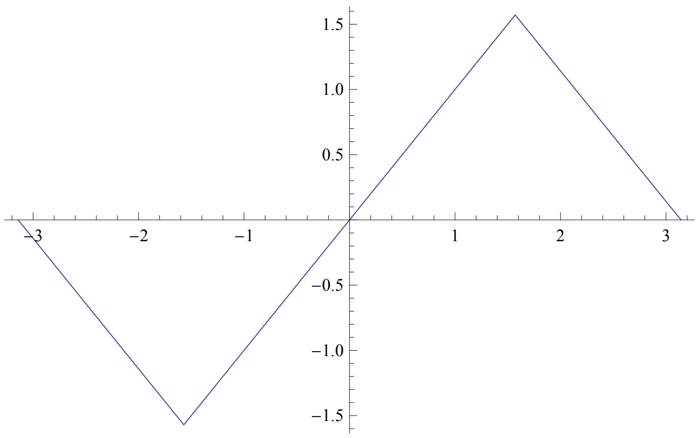
```
Plot[{f[t], fr[t, 7]}, {t, -2, 2}]
```




```
Plot[{f[t], fr[t, 3], fr[t, 7]}, {t, -2, 2}]
```




```
(* b *)

f[x_] := (-x - Pi) (1 - UnitStep[x + Pi / 2]) +
  x ( UnitStep[x + Pi / 2] - UnitStep[x - Pi / 2]) + (-x + Pi) UnitStep[x - Pi / 2]

Plot[f[x], {x, -Pi, Pi}]
```



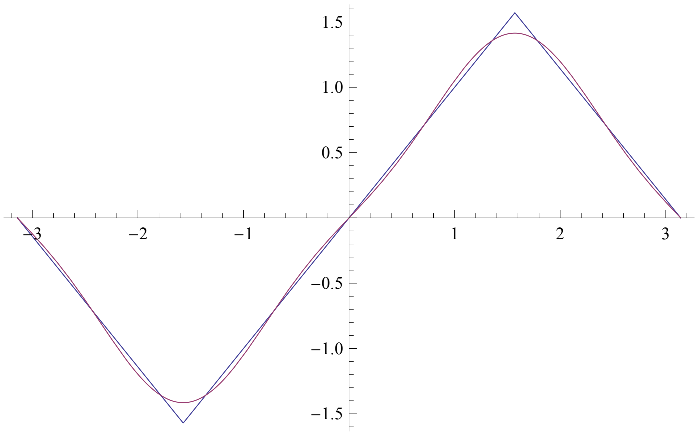
```
(* die Periode: *)

p = 2 Pi;

k = 2 Pi / p
```

1

```
(* Fourier-Reihe bis zur 3. Ordnung, also bis zur 2. Oberschwingung *)

fr[x_, 3] = FourierTrigSeries[f[x], x, 3, FourierParameters → {1, k}]
```

$$\frac{4\,\mathrm{Sin}[x]}{\pi} - \frac{4\,\mathrm{Sin}[3\,x]}{9\,\pi}$$

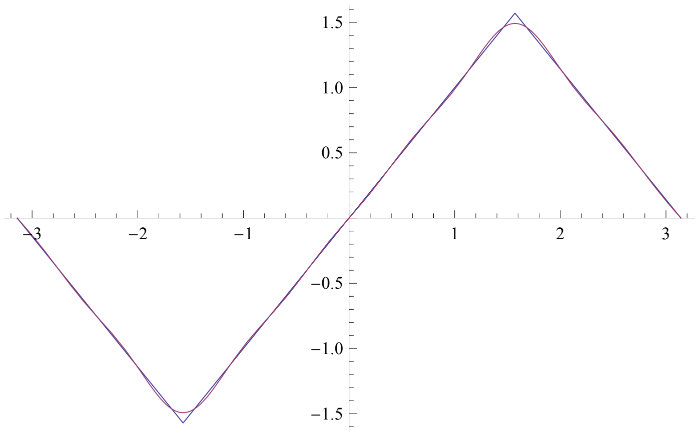
```
Plot[{f[x], fr[x, 3]}, {x, -Pi, Pi}]
```



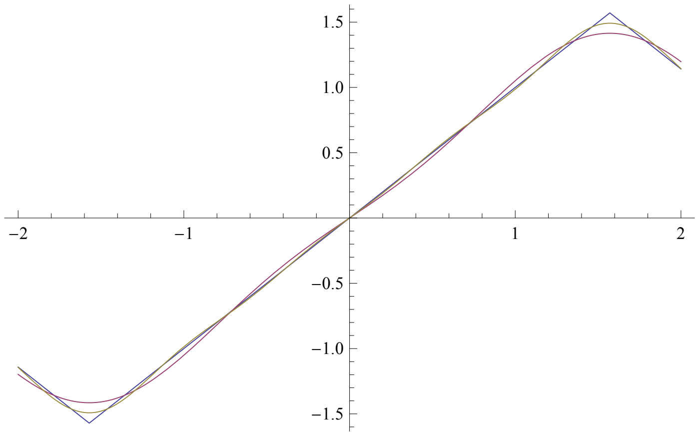

```
(* Fourier-Reihe bis zur 7. Ordnung, also bis zur 6. Oberschwingung *)

fr[x_, 7] = FourierTrigSeries[f[x], x, 7, FourierParameters → {1, k}]
```

$$\frac{4\,\mathrm{Sin}[x]}{\pi} - \frac{4\,\mathrm{Sin}[3\,x]}{9\,\pi} + \frac{4\,\mathrm{Sin}[5\,x]}{25\,\pi} - \frac{4\,\mathrm{Sin}[7\,x]}{49\,\pi}$$

```
Plot[{f[t], fr[t, 7]}, {t, -Pi, Pi}]
```



```
Plot[{f[t], fr[t, 3], fr[t, 7]}, {t, -2, 2}]
```



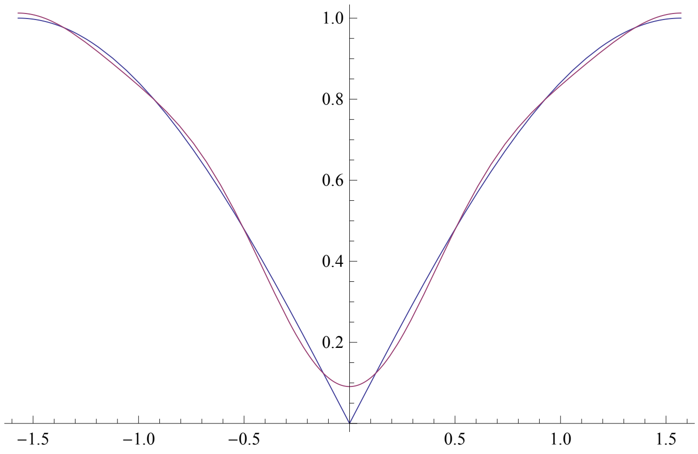

```
(* c *)

f[t_] := Abs[Sin[t]]

(* die Periode: *)

p = Pi;

ω = 2 Pi / p
```

2

```
(* Fourier-Reihe bis zur 3. Ordnung, also bis zur 2. Oberschwingung *)

fr[t_, 3] = FourierTrigSeries[f[t], t, 3, FourierParameters → {1, ω}]
```

$$\frac{2}{\pi} - \frac{4 \operatorname{Cos}[2\,t]}{3\,\pi} - \frac{4 \operatorname{Cos}[4\,t]}{15\,\pi} - \frac{4 \operatorname{Cos}[6\,t]}{35\,\pi}$$

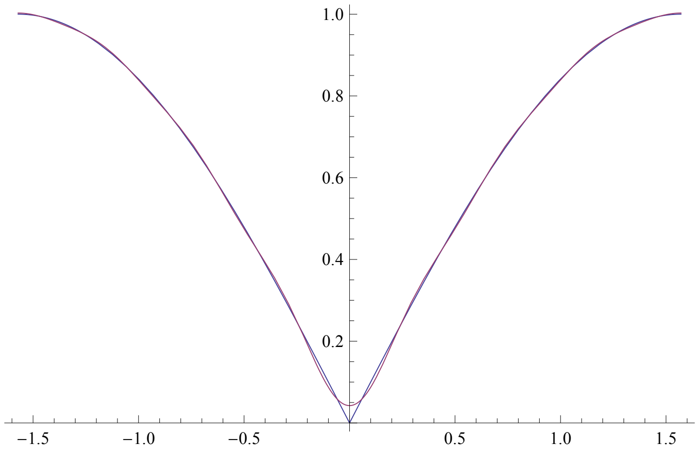
```
Plot[{f[t], fr[t, 3]}, {t, -Pi / 2, Pi / 2}]
```



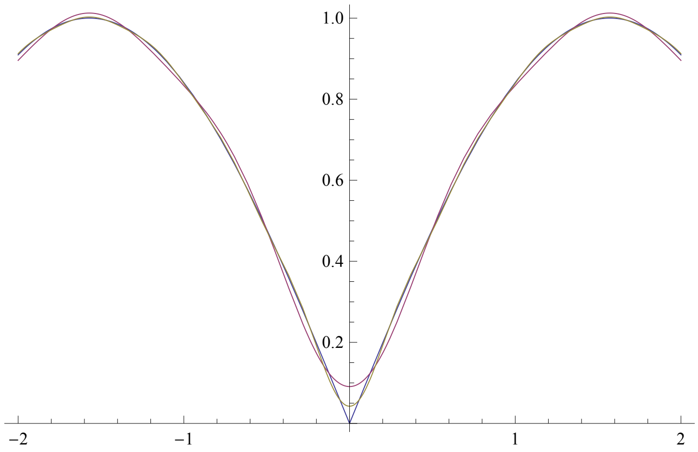
```
(* Fourier-Reihe bis zur 7. Ordnung, also bis zur 6. Oberschwingung *)

fr[t_, 7] = FourierTrigSeries[f[t], t, 7, FourierParameters → {1, ω}]
```

$$\frac{2}{\pi} - \frac{4\,\mathrm{Cos}[2\,t]}{3\,\pi} - \frac{4\,\mathrm{Cos}[4\,t]}{15\,\pi} - \frac{4\,\mathrm{Cos}[6\,t]}{35\,\pi} - \frac{4\,\mathrm{Cos}[8\,t]}{63\,\pi} - \frac{4\,\mathrm{Cos}[10\,t]}{99\,\pi} - \frac{4\,\mathrm{Cos}[12\,t]}{143\,\pi} - \frac{4\,\mathrm{Cos}[14\,t]}{195\,\pi}$$

```
Plot[{f[t], fr[t, 7]}, {t, -Pi / 2, Pi / 2}]
```

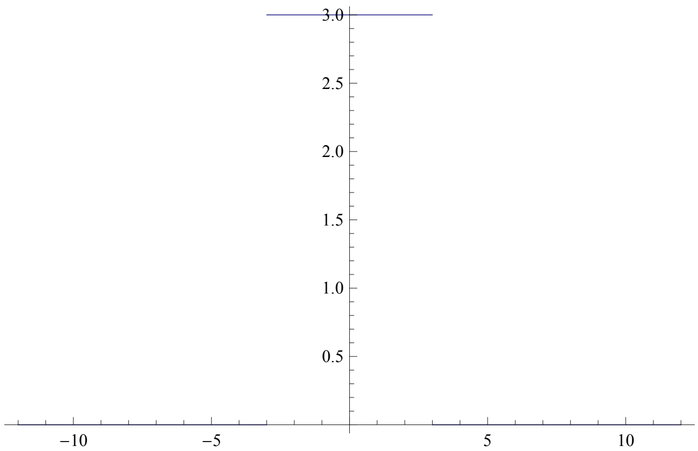


```
Plot[{f[t], fr[t, 3], fr[t, 7]}, {t, -2, 2}]
```



```
(* 3 *)

In[126]:= Remove["Global`*"]

In[127]:= f[t_] := f0 (UnitStep[t + T / 2] - UnitStep[t - T / 2])

In[128]:= T = 6

Out[128]= 6

In[129]:= f0 = 3

Out[129]= 3

In[130]:= Plot[f[t], {t, -2 T, 2 T}]
```

Out[130]=




```
In[131]:= Clear[f0, T]

In[133]:= Integrate[f[t] * Exp[- I * ω * t],
           {t, -∞, ∞}, Assumptions → {t ∈ Reals, ω ∈ Reals, T > 0}]
```

Out[133]= $\frac{2\, f0\, \mathrm{Sin}\left[\frac{T\,\omega}{2}\right]}{\omega}$

```
(* Wir können ja auch mit dem folgenden Integral arbeiten: *)

Integrate[f[t] * Exp[- I * ω * t], {t, -T / 2, T / 2}, Assumptions → T > 0]
```

$\frac{2\, f0\, \mathrm{Sin}\left[\frac{T\,\omega}{2}\right]}{\omega}$

```
(* mit dem eingebauten Befehl: *)
```

```
F[ω_] = FourierTransform[f[t], t, ω, FourierParameters → {1, -1}]
```

$$\frac{2\ f0\ \mathrm{Sin}\left[\frac{T\,\omega}{2}\right]}{\omega}$$

```
ω1 = 2 Pi / T
```

$$\frac{2\,\pi}{T}$$

```
(* Mathematica kann die gewünschte Integration scheinbar exakt;
das Ergebnis ist eine eingebaute Funktion,
die auch nur näherungsweise berechnet werden kann *)
Integrate[F[ω], {ω, -ω1, ω1}]
```

4 f0 SinIntegral[π]

```
N[%, 4]
```

7.408 f0

```
(* Taylorentwicklung des Integranden bis zur Ordnung n : *)
n = 2
```

2

```
tp[ω_] = Series[F[ω], {ω, 0, n}] // Normal
```

$$f0\ T - \frac{1}{24}\ f0\ T^3\ \omega^2$$

```
Integrate[tp[ω], {ω, -ω1, ω1}]
```

$$4\ f0\ \pi - \frac{2\ f0\ \pi^3}{9}$$

```
% // N
```

5.67609 f0

```
n = 12
```

12

```
tp[ω_] = Series[F[ω], {ω, 0, n}] // Normal
```

$$f0\ T - \frac{1}{24}\ f0\ T^3\ \omega^2 + \frac{f0\ T^5\ \omega^4}{1920} - \frac{f0\ T^7\ \omega^6}{322\,560} + \frac{f0\ T^9\ \omega^8}{92\,897\,280} - \frac{f0\ T^{11}\ \omega^{10}}{40\,874\,803\,200} + \frac{f0\ T^{13}\ \omega^{12}}{25\,505\,877\,196\,800}$$

```
Integrate[tp[ω], {ω, -ω1, ω1}]
```

$$4\ f0\ \pi - \frac{2\ f0\ \pi^3}{9} + \frac{f0\ \pi^5}{150} - \frac{f0\ \pi^7}{8820} + \frac{f0\ \pi^9}{816\,480} - \frac{f0\ \pi^{11}}{109\,771\,200} + \frac{f0\ \pi^{13}}{20\,237\,817\,600}$$

```
N[%, 4]
```

7.408 f0